AP Calculus Testbank
(Chapter 7)
(Mr. Surowski)

Part I. Multiple-Choice Questions

1. Suppose that a function $y = f(x)$ is given with $f(x) \geq 0$ for $0 \leq x \leq 4$. If the area bounded by the curves $y = f(x)$, $y = 0,\ x = 0$, and $x = 4$ is revolved about the x-axis, then the volume of the resulting solid would best be computed by the method of

   (A) disks/washers (B) shells (C) known cross sections.

2. Suppose that a function $y = f(x)$ is given with $f(x) \geq 0$ for $0 \leq x \leq 4$. If the area bounded by the curves $y = f(x)$, $y = 0,\ x = 0$, and $x = 4$ is revolved about the x-axis, then the volume of the solid of revolution is given by

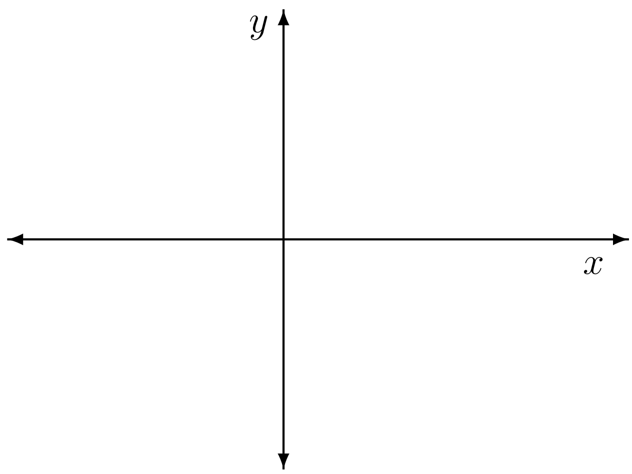
   (A) $2\pi \int_0^4 x f(x)^2\, dx$

   (B) $2\pi \int_0^4 x^2 f(x)\, dx$

   (C) $2\pi \int_0^4 \sqrt{1 + f(x)^2}\, dx$

   (D) $\pi \int_0^4 f(x)^2\, dx$

   (E) $2\pi \int_0^4 f(x)^2\, dx$



3. A parabola is drawn having focus $(0, 2)$ and directrix $y = 4$. The definite integral representing the arc length of that portion of the parabola on or above the x-axis is given by

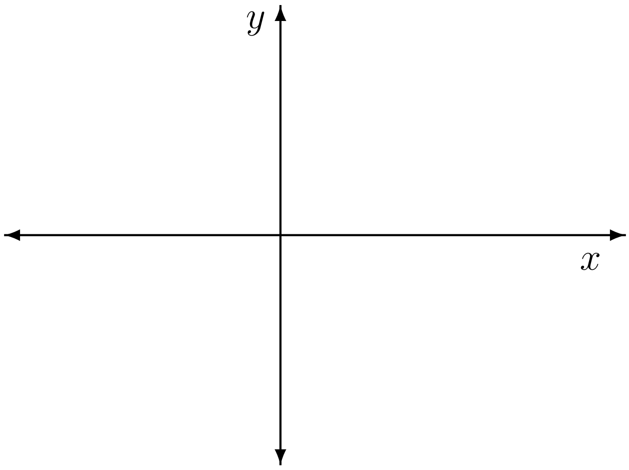
(A) $\displaystyle\int_{0}^{12} \sqrt{4 - x^2}\, dx$

(B) $\displaystyle\int_{0}^{2\sqrt{3}} \sqrt{4 - x^2}\, dx$

(C) $\displaystyle\int_{0}^{2\sqrt{3}} \sqrt{4 + x^2}\, dx$

(D) $\displaystyle\int_{0}^{12} \sqrt{4 + x^2}\, dx$

(E) $\displaystyle\int_{0}^{12} \frac{dx}{\sqrt{4 + x^2}}$



4. Consider the solid of revolution formed by revolving the area bounded by the curve $y = 1/x$, the x-axis, the line $x = 1$ and the line $x = a,\ \ (a > 1)$ about the x-axis. The integral representing the volume of this solid is

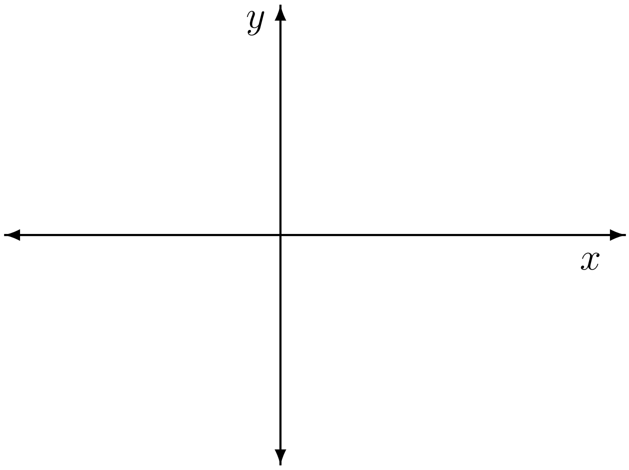
(A) $\displaystyle\pi \int_{1}^{a} \frac{dx}{x}$

(B) $\displaystyle 2\pi \int_{1}^{a} \frac{dx}{x}$

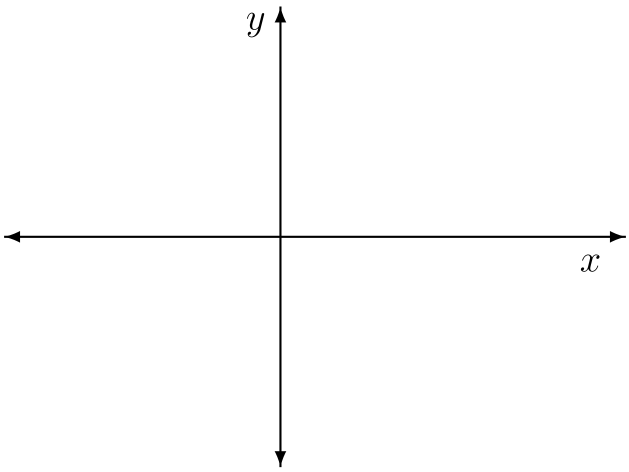
(C) $\displaystyle\pi \int_{1}^{a} \frac{dx}{x^2}$

(D) $\displaystyle 2\pi \int_{1}^{a} \frac{dx}{x^2}$

(E) $\displaystyle\pi \int_{1}^{a} \frac{dx}{\sqrt{x}}$



5. Consider the surface of revolution formed by revolving the the curve $y = 1/x,\ 1 \le x \le a$ about the x-axis. Then the surface area is given by the definite integral



(A) $2\pi \displaystyle\int_1^a \frac{dx}{x}$

(B) $2\pi \displaystyle\int_1^a \frac{dx}{x^2}$

(C) $2\pi \displaystyle\int_1^a \frac{\sqrt{1+x^4}\,dx}{x^3}$

(D) $2\pi \displaystyle\int_1^a \left(1+\frac{1}{x^2}\right)\,dx$

(E) $2\pi \displaystyle\int_1^a \sqrt{1+\frac{1}{x^4}}\,dx$

6. Which of the following integrals correctly gives the area of the region consisting of all points above the x-axis and below the curve $y = 8 + 2x - x^2$?

(A) $\displaystyle\int_{-2}^{4} (x^2 - 2x - 8)\,dx$

(B) $\displaystyle\int_{-4}^{2} (8 + 2x - x^2)\,dx$

(C) $\displaystyle\int_{-2}^{4} (8 + 2x - x^2)\,dx$

(D) $\displaystyle\int_{-4}^{2} (x^2 - 2x - 8)\,dx$

(E) $\displaystyle\int_{2}^{4} (8 + 2x - x^2)\,dx.$

7. A solid is generated with the region in the first quadrant bounded by the graph of $y = 1 + \sin^2 x$, the line $x = \frac{\pi}{2}$, the x-axis, and the y-axis is revolved about the x-axis. Its volume is found by evaluating which of the following integrals?

(A) $\pi \int_0^1 (1 + \sin^4 x)\, dx$

(B) $\pi \int_0^1 (1 + \sin^2 x)^2\, dx$

(C) $\pi \int_0^{\frac{\pi}{2}} (1 + \sin^4 x)\, dx$

(D) $\pi \int_0^{\frac{\pi}{2}} (1 + \sin^2 x)^2\, dx$

(E) $\pi \int_0^{\frac{\pi}{2}} (1 + \sin^2 x)\, dx$.

8. The volume generated by revolving about the x-axis the region above the curve $y = x^3$, below the line $y = 1$, and between $x = 0$ and $x = 1$ is

(A) $\frac{\pi}{42}$ (B) 0.143π (C) $\frac{\pi}{7}$ (D) 0.643π (E) $\frac{6\pi}{7}$

9. Find the distance traveled (to three decimal places) from $t = 1$ to $t = 5$ seconds, for a particle whose velocity is given by $v(t) = t + \ln t$.

(A) 6.000 (B) 1.609 (C) 16.047 (D) 0.800 (E) 148.413

10. Find the area of the region bounded by the parabolas $y = x^2$ and $y = 6x - x^2$

(A) 9 (B) 27 (C) 6 (D) -9 (E) -18

11. What is the area of the region in the first quadrant enclosed by the graph of $y = e^{-\frac{x^2}{4}}$ and the line $y = 0.5$?

(A) 0.240 (B) 0.516 (C) 0.480 (D) 1.032 (E) 1.349

12. The base of a solid S is the region enclosed by the graph of $4x + 5y = 20$, the x-axis, and the y-axis. If the cross-sections of S perpendicular to the x-axis are semicircles, then the volume of S is

(A) $\frac{5\pi}{3}$ (B) $\frac{10\pi}{3}$ (C) $\frac{50\pi}{3}$ (D) $\frac{225\pi}{3}$ (E) $\frac{425\pi}{3}$

13. The volume of the solid that results when the area between the curve $y = e^x$ and the line $y = 0$, from $x = 1$ to $x = 2$, is revolved around the x-axis is

(A) $2\pi(e^4 - e^2)$ (B) $\frac{\pi}{2}(e^4 - e^2)$ (C) $\frac{\pi}{2}(e^2 - e)$ (D) $2\pi(e^2 - e)$ (E) $2\pi e^2$

14. What is the volume of the solid generated by rotating about the y-axis the region enclosed by $y = \sin x$ and the x-axis, from $x = 0$ to $x = \pi$?

(A) π^2 (B) $2\pi^2$ (C) $4\pi^2$ (D) 2 (E) 4

Part II. Free-Response Questions

1. Given the velocity function $v(t) = \dfrac{t-1}{4+t^2},\ 0 \le t \le 2,\ x(0) = 0,$,

   (a) determine the terminal position of the particle, and

   (b) determine the total distance traveled by the particle.

2. Given the velocity function $v(t) = 1 + 2\sin t,\ 0 \le t \le 11\pi/6,$, where $x(0) = 0$,

   (a) determine the terminal position of the particle, and

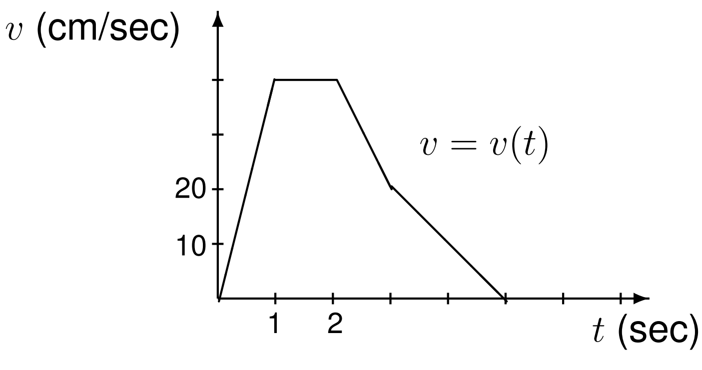
   (b) determine the total distance traveled by the particle.

3. Given the velocity function $v(t) = t\cos \pi t,\ 0 \le t \le 1.5$, where $x(0) = 0$,

   (a) determine the terminal position of the particle, and

   (b) determine the total distance traveled by the particle.

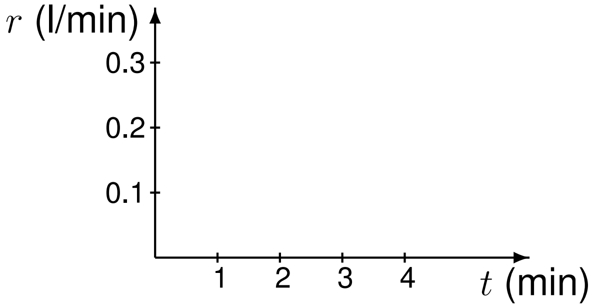
4. The velocity function of a particle has the graph depicted below. Find the total distance traveled by the particle over the first five seconds.



5. Suppose that a particle is initially at rest at the origin, but at time $t = 0$ a force is applied to the particle which results in an acceleration of +10 cm/sec^2. Locate this particle on the x-axis after 5 seconds.

6. Water is flowing from a faucet into a one-litre bottle at a rate of $r(t) = te^{-2t}$ l/min. After 2 minutes the water is turned up to a constant rate of of .2 l/min.

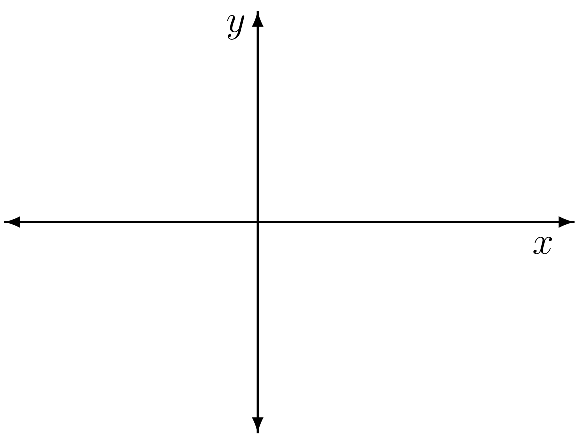
(a) Graph the function $r = r(t)$ depicting the rate of flow of water.



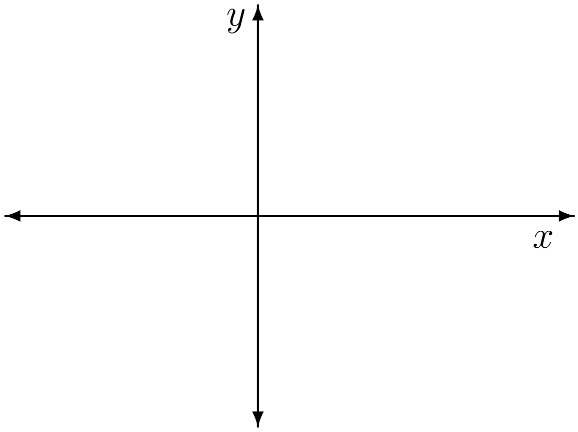
(b) Will the bottle be full after 4 minutes?

7. Suppose that a particle is resting at the origin and that a force of $F = F(t)$ cm/sec^2, $t \geq 0$, is applied to the particle over the interval $0 \leq t < \infty$. Assuming that $F(t) > 0$ over this interval, compute $\lim_{t \to \infty} x(t)$ and justify your answer.

8. Graph the region bounded by the curves $x = y^2$ and $x + 2y^2 = 3$ and compute its area.

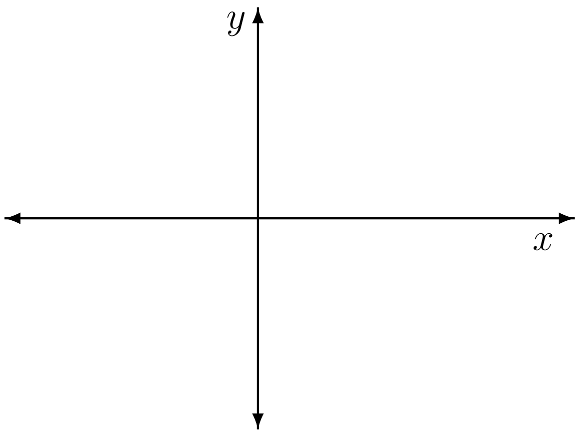


9. Graph the region bounded by the curves $y = -x^2 + 3x$ and $y = 2x^3 - x^2 - 5x$ and compute its area.



10. Set up an integral (without evaluating it) that will compute the area of the region

$$\frac{x^2}{9} + \frac{y^2}{4} \leq 1, \quad -x \leq y \leq x.$$

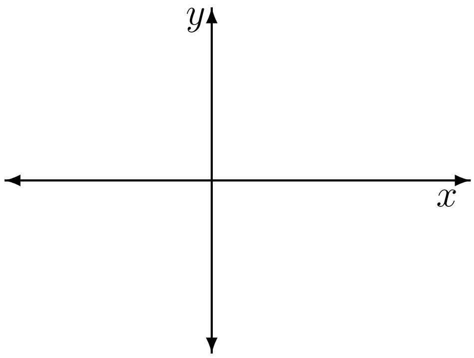


11. Consider the region bounded by the curve $y = 1/x^p,\ x = 1,\ x = a$, and the x-axis.

(a) Compute the volume of the solid of revolution obtained by revolving the above region about the x-axis.

(b) If $V(a)$ represents the volume given in part (a) above, compute $\lim_{a \to \infty} V(a)$.

(c) There is a value p_0 such that if $p \leq p_0$, the limit in part (b) above is infinite and if $p > p_0$, the limit in part (b) above is finite. Find this number p_0.

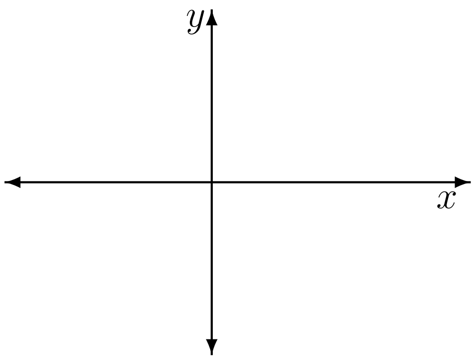


12. Consider the region bounded by the curve $y = 1/x^p,\ x = 1,\ x = a$, and the x-axis.

(a) Compute the volume of the solid of revolution obtained by revolving the above region about the y-axis.

(b) If $V(a)$ represents the volume given in part (a) above, compute $\lim_{a \to \infty} V(a)$.

(c) There is a value p_0 such that if $p \leq p_0$, the limit in part (b) above is infinite and if $p > p_0$, the limit in part (b) above is finite. Find this number p_0.



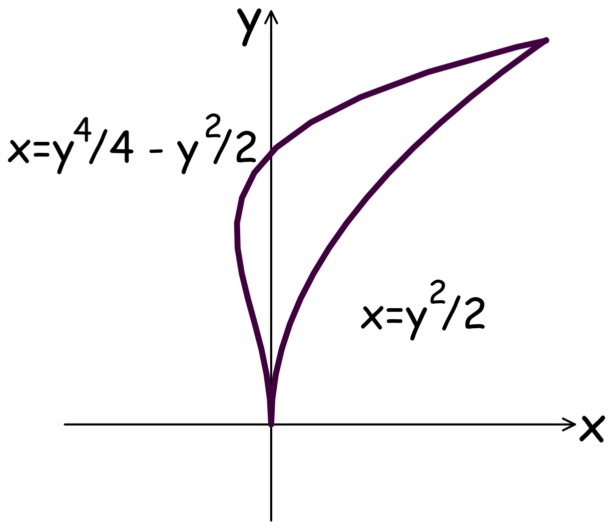
13. Consider the surface generated by revolving the curve $y = 1/x^p$, $1 \leq x \leq a$, about the x-axis.

    (a) Express the area of the above surface as an integral (you probably won't be able to evaluate this integral).

    (b) Show that if $p \leq 1$, then $\lim_{a \to \infty} S(a) = +\infty$.

    (c) (This is harder) Show that if $p > 1$, then $\lim_{a \to \infty} S(a)$ is finite.

14. The region below is revolved about the x-axis to form a solid of revolution. Find the volume of this solid.



15. A solid object has a flat base formed by the region enclosed by the parabola with focus having coordinates $(0, 2)$ and directrix $y = 4$ and by the x-axis. Each cross section is an equilateral triangle perpendicular to the base and parallel to the directrix. Compute the volume of this object.

16. Compute the length of that section of the curve $x = y^4/4 + 1/8y^2$ that joins $(3/8, 1)$ to the point $(129/32, 2)$.

17. If $S(a)$ represents the surface of revolution of problem 5, (multiple-choice section) compute $\lim_{a\to\infty} S(a)$.

18. Use integral calculus to show that the volume of a right circular cone of height h and base area A is $\frac{1}{3}Ah$.

19. Suppose that a metal chain weighing 1000 newtons/m is hanging over a building. Assuming that the building is 30 m tall, and that the chain is just touching the ground, what is the total work required to pull the chain onto the top of the building?

20. Suppose that an object rests at the point $x = 0$ on the x-axis. We then start pushing this box in the positive x direction, giving the box a speed of $e^{-t/2}$ m/sec. Assume that there is a force due to friction, the magnitude of which is 1/10 the speed of the box. Find the total work needed to push the box for 10 seconds.

21. Assume that there is a heavy box sitting outside on the pavement. We are going to move this box a total of 20 feet by sliding it along the pavement. The relevant force here is that of friction, which we shall assume is proportional to the speed at which we slide the box. Which will result in less work, sliding the box quickly over the necessary 20 feet or sliding it slowly? Please explain.